Calculations Policy

This policy was adopted/updated:	**July 2016**
This policy will be reviewed:	**Summer 2017**
Governor Committee Responsibility:	
Statutory policy:	**No**

Introduction

At the centre of the mastery approach to the teaching of mathematics is the belief that all pupils have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly with calculation strategies, pupils must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations.

This policy outlines the different calculation strategies that should be taught and used in KS1, in line with the requirements of the 2014 Primary National Curriculum.

Background

The 2014 Primary National Curriculum for mathematics differs from its predecessor in many ways. Alongside the end of Key Stage year expectations, there are suggested goals for each year; there is also an emphasis on depth before breadth and a greater expectation of what pupils should achieve. In addition, there is a whole new assessment method, as the removal of levels gives schools greater freedom to develop and use their own systems.

One of the key differences is the level of detail included, indicating what pupils should be learning and when. This is suggested content for each year group, but schools have been given autonomy to introduce content earlier or later, with the expectation that by the end of each key stage the required content has been covered.

For example, in Year 2, it is suggested that pupils should be able to 'add and subtract one-digit and two-digit numbers to 20, including zero' and a few years later, in Year 5, they should be able to 'add and subtract whole numbers with more than four digits, including using formal written methods (columnar addition and subtraction)'.

In many ways, these specific objectives make it easier for teachers to plan a coherent approach to the development of pupils' calculation skills. However, the expectation of using formal methods is rightly coupled with the explicit requirement for pupils to use concrete materials and create pictorial representations – a key component of the mastery approach.

Purpose

The purpose of this policy is twofold. Firstly, it makes teachers aware of the strategies that pupils are formally taught within each year group that will support them to perform mental and written calculations. Secondly, it supports teachers in identifying appropriate pictorial representations and concrete materials to help develop understanding.

The policy only details the strategies; teachers must plan opportunities for pupils to apply these; for example, when solving problems, or where opportunities emerge elsewhere in the curriculum.

How to use the policy

For ease of reference, the strategies and examples contained in this policy are cross-referenced with objectives from the *2014 Maths Programme of Study*. For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. Please note that the concrete and representation examples are not exhaustive, and teachers and pupils may well come up with alternatives.
Where necessary, additional guidance is given to support in teaching the given strategies.
Please note that the principle of the concrete-pictorial-abstract (CPA) approach is that for pupils to have a true understanding of a mathematical concept, they need to master all three phases. Reinforcement is achieved by going back and forth between these representations. For example, if a child has moved on from the concrete to the pictorial, it does not mean that the concrete cannot be used alongside the pictorial. Or if a child is working in the abstract, 'proving' something or 'working out' could involve use of the concrete or pictorial. In short, these are not always 'exclusive' representations.

Mathematical language

The 2014 National Curriculum is explicit in articulating the importance of pupils using the correct mathematical language as a central part of their learning. Indeed, in certain year groups, the non-statutory guidance highlights the requirement for pupils to extend their language around certain concepts.

> **"The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof."**
> ***2014 Maths Programme of Study***

It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate mathematical vocabulary.
New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct.

✓	x
ones	units
is equal to	equals
zero	oh (the letter O)

Exemplification

You will see that throughout this document, calculations are presented in a variety of ways, It is important for pupils' mathematical understanding to experience and work with calculations and missing numbers in different positions relative to the = symbol. Examples used in classwork and independent work should reflect this.

Estimation

Pupils are expected to use their developing number sense from Year 1 to make predictions about the answers to their calculations. As their range of mental strategies increases, these predictions and, later, estimates should become increasingly sophisticated and accurate. All teaching of calculation should emphasise the importance of making and using these estimates to check, first, the sense and, later, the accuracy of their calculations.

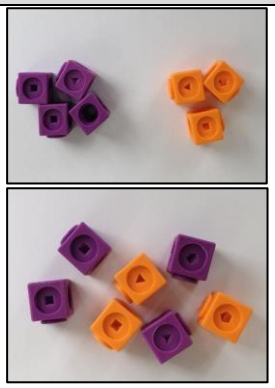
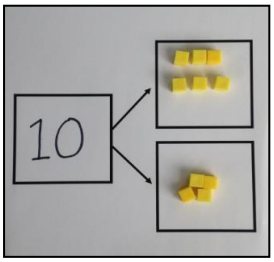
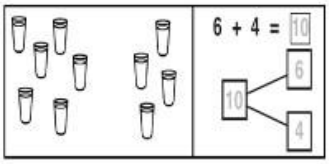
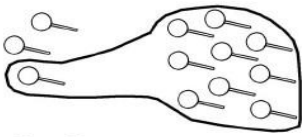
Progression in calculations: Year 1

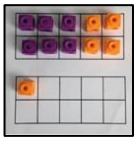
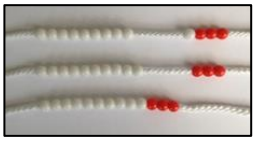
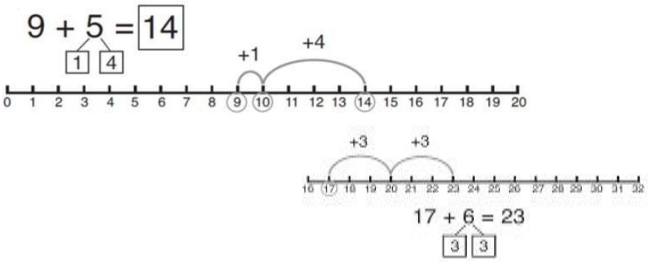
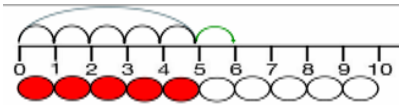
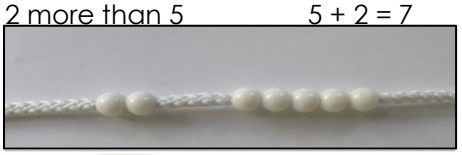
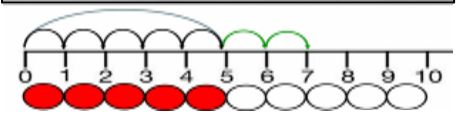
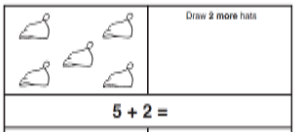
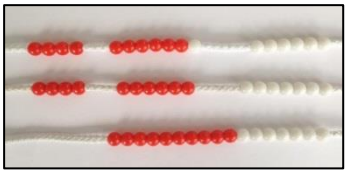
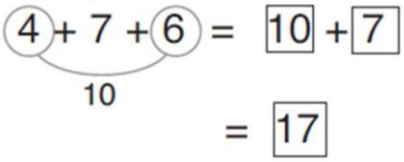
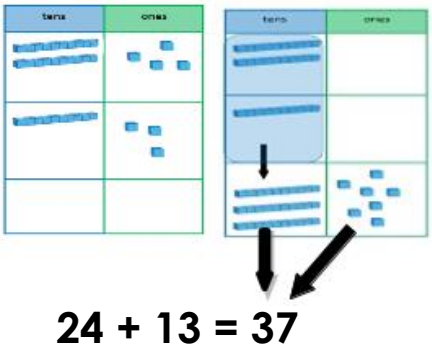
National Curriculum objectives linked to addition and subtraction

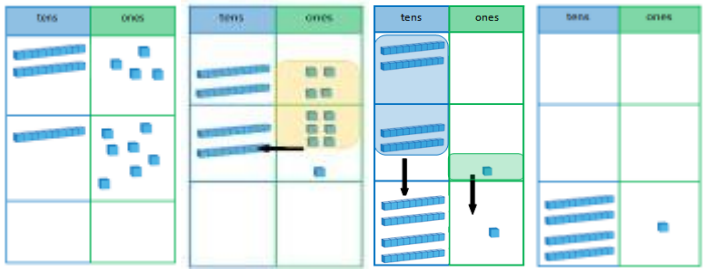
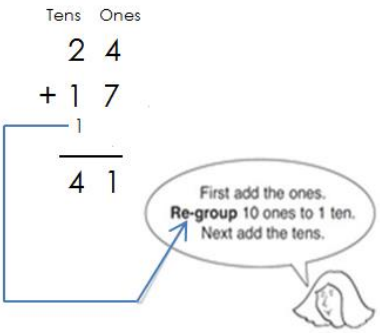
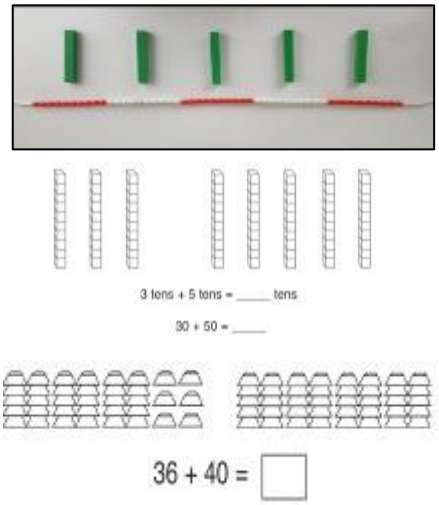
These objectives are explicitly covered through the strategies outlined in this document:

- Add and subtract one-digit and two-digit numbers to 100, including zero (N.B. Year 1 N.C. objective is to do this with numbers to 20).
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones, a two-digit number and tens, 2 two-digit numbers; add 3 one-digit numbers (Year 2).
- Represent and use number bonds and related subtraction facts within 20.
- Given a number, identify 1 more and 1 less.
- Show that addition of two numbers can be done in any order (commutative) but subtraction of one number from another cannot (Year 2).
- Recognise the inverse relationship between addition and subtraction and use this to solve missing number problems (Year 2).
- The following objectives should be planned for lessons where new strategies are being introduced and developed:
- Read, write and interpret mathematical statements involving addition (+), subtraction (−) and equal (=) signs.
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems, such as
- $7 = \square - 9$.
- Solve problems with addition and subtraction:
- Using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- Applying their increasing knowledge of mental methods

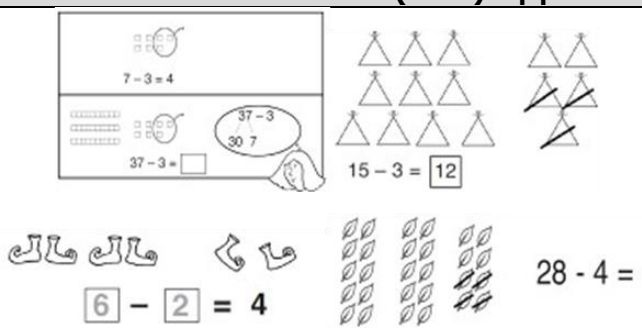
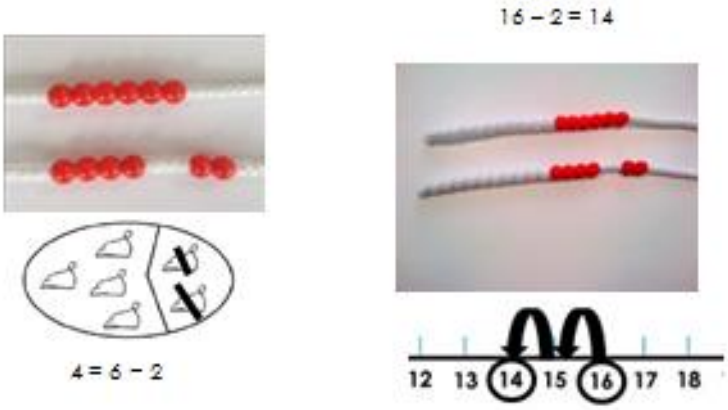
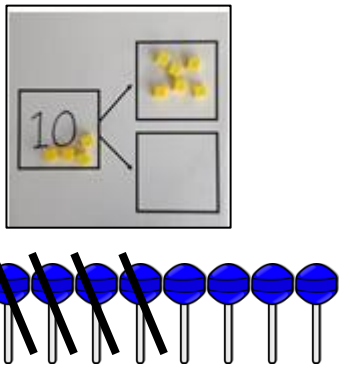
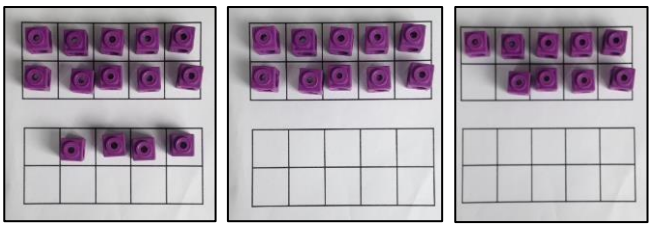
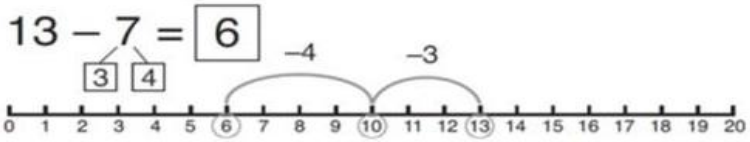
ADDITION – Year 1

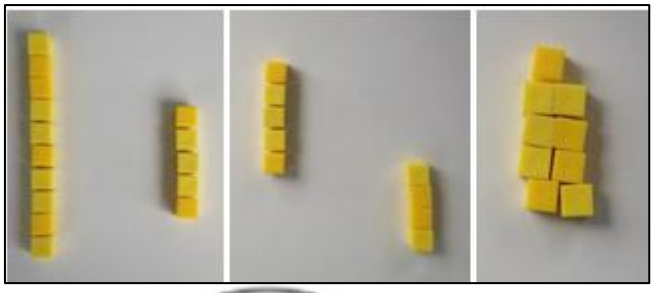
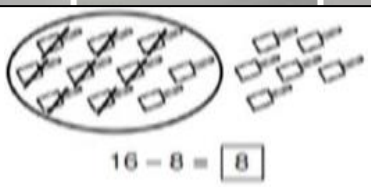
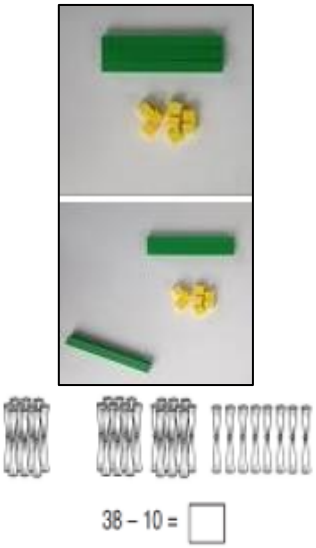
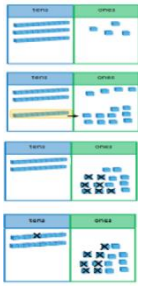
	Strategy and Guidance	Concrete Pictorial Abstract (CPA) Approaches
Step 1	**Joining two groups** and then recounting all objects using one-to-one correspondence.	 4 + 3 = 7   5 + 3 = 8
Step 2	**Counting on** As a strategy, this should be limited to adding small quantities only (1, 2 or 3) with pupils understanding that counting on from the greater is more efficient. Pupils should be encouraged to rely on number bonds knowledge as time goes on, rather than using counting on as their main strategy.	  8 + 1 = 9   12 + 3 = 15
Step 3	**Part-part-whole** Teach both addition and subtraction alongside each other, as pupils will use this model to identify the inverse link between them. Pupils could place ten on top of the whole as well as writing it down. The parts could also be written in alongside the concrete representation. This model begins to develop the understanding of the commutativity of addition, as pupils become aware that the parts will make the whole in any order.	 10 = 6 + 4 10 − 6 = 4 10 − 4 = 6 10 = 4 + 6 
Step 4	**Regrouping ten ones to make ten** This is an essential skill that will support column addition later on.	 3 + 9 = 12 

Step 5	**'Make ten' strategy** Pupils should be encouraged to start at the bigger number and use the smaller number to make ten. The colours of the beads on the bead string make it clear how many more need to be added to make ten. Also, the empty spaces on the ten frame make it clear how many more are needed to make ten.	6 + 5 = 11  4 + 9 = 13  
Step 6	**Adding 1, 2, 3 more** Here the emphasis should be on the language rather than the strategy. As pupils are using the beadstring, ensure that they are explaining using language such as; '1 more than 5 is equal to 6.' '2 more than 5 is 7.' '8 is 3 more than 5.'	1 more than 5 5 + 1 = 6  2 more than 5 5 + 2 = 7   
Step 7	**Adding three single digit numbers (make ten first)** Pupils may need to try different combinations before they find the two numbers that make 10. The first bead string shows 4, 7 and 6. The colours of the bead string show that it makes more than ten. The second bead string shows 4, 6 and then 7. The final bead string shows how they have now been put together to find the total.	 
Step 8	**Partitioning to add (no regrouping)** Place value grids and Dienes blocks should be used as shown in the diagram before moving onto the pictorial representations. Dienes blocks should always be available, as the main focus in Year 1 is the concept of place value rather than mastering the procedure. When not regrouping, partitioning is a mental strategy and does not need formal recording in columns. This representation prepares them for using column addition with formal recording.	24 + 13 = 37 

Step 9	**Introducing column method for addition, regrouping only** Dienes blocks and place value grids should be used as shown in the diagrams. Even when working pictorially, pupils should have access to Dienes blocks. See additional guidance on unit pages for extra guidance on this strategy.	24 + 17  
Step 10	**Adding multiples of ten** Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important, as pupils need to understand that it is a **ten** and not a one that is being added. It also emphasises the link to known number facts. E.g. '2 + 3 is equal to 5. So 2 tens + 3 tens is equal to 5 tens.	**50 = 30 + 20** 

SUBTRACTION – Year 1

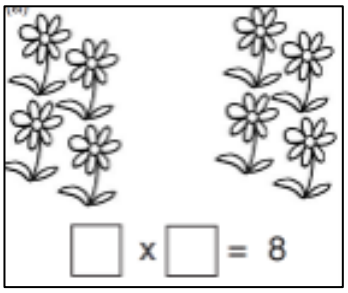
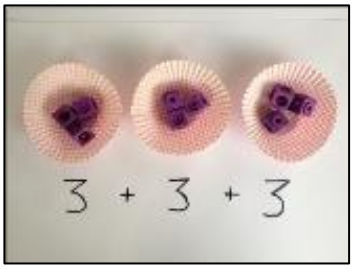
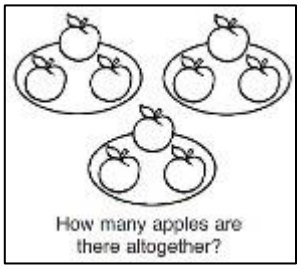
	Strategy and Guidance	Concrete Pictorial Abstract (CPA) Approaches
Step 1	**Taking away from the ones** When this is first introduced, the concrete representation should be based upon the diagram. Real objects should be placed on top of the images as one-to-one correspondence so that pupils can take them away, progressing to representing the group of ten with a tens rod and ones with ones cubes.	
Step 2	**Counting back** Subtracting 1, 2, or 3 by counting back Pupils should be encouraged to rely on number bonds knowledge as time goes on, rather than using counting back as their main strategy.	
Step 3	**Part-part-whole** Teach both addition and subtraction alongside each other, as the pupils will use this model to identify the link between them. Pupils start with ten cubes placed on the whole. They then remove what is being taken away from the whole and place it on one of the parts. The remaining cubes are the other part and also the answer. These can be moved into the second part space.	**10 – 6 = 4** 
Step 4	**Make ten strategy** single digit number from a 2-digit number Pupils identify how many need to be taken away to make ten first. Then they take away the rest to reach the answer.	**14 – 5 = 9**  
Step 5	**Regroup a ten into 10 ones** After the initial introduction, the Dienes blocks should be placed on a place value chart to support place value understanding. This will support pupils when they later use the column method.	  20 – 4 =

Step 6	**Taking away from the tens** Pupils should identify that they can also take away from the tens and get the same answer. This reinforces their knowledge of number bonds to 10 and develops their application of number bonds for mental strategies.	  16 – 8 = 8
Step 7	**Partitioning to subtract without regrouping** Dienes blocks on a place value chart (developing into using images on the chart) should be used, as when adding 2-digit numbers, reinforcing the main concept of place value for Year 1. When not regrouping, partitioning is a mental strategy and does not need formal recording in columns. This representation prepares them for using column subtraction with formal recording. See additional guidance on unit pages to support with this strategy.	**34 - 13 = 21**  **34 – 13 = 21**
Step 8	**Subtracting multiples of ten** *Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important as pupils need to understand that it is a* ***ten*** *not a one that is being taken away.*	**40 = 60 – 20**  **38 – 10 = 28**  38 – 10 =
Step 9	**Column method with regrouping** *This example shows how pupils should work practically when being introduced to this method. There is no formal recording in columns in Year 1 but this practical work will prepare pupils for formal methods in Year 2.* *See additional guidance on unit pages to support with this method.*	**34 – 17 = 17** 

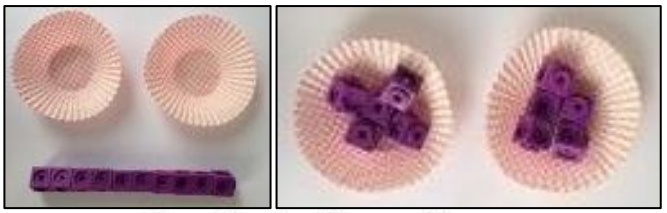
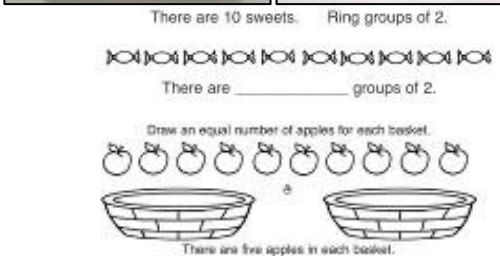
National Curriculum objectives linked to multiplication and division

These objectives are explicitly covered through the strategies outlined in this document:

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

MULTIPLICATION – Year 1		
	Strategy and Guidance	**Concrete Pictorial Abstract (CPA) Approaches**
Step 1	**Skip counting in multiples of 2, 5, 10 from zero** The representation for the amount of groups supports pupils' understanding of the written equation. So two groups of 2 are 2, 4. Or five groups of 2 are 2, 4, 6, 8, 10. Count the groups as pupils are skip counting. Number lines can be used in the same way as the bead string. Pupils can use their fingers as they are skip counting.	 **4 × 5 = 20**  **2 × 4 = 8**
Step 2	**Making equal groups and counting the total** How this would be represented as an equation will vary. This could be 2 × 4 or 4 × 2. The importance should be placed on the vocabulary used alongside the equation. So this picture could represent 2 groups of 4 or 4 twice.	  Draw  to show 2 x 3 = 6
Step 3	**Solve multiplications using repeated addition**	  **3 + 3 + 3 = 9**

DIVISION – Year 1

	Strategy and Guidance	Concrete Pictorial Abstract (CPA) Approaches
Step 1	**Sharing objects into groups** Pupils should become familiar with division equations through working practically. The division symbol is not formally taught at this stage.	**10 ÷ 2 = 5**  

Progression in calculations: Year 2

National Curriculum objectives linked to addition and subtraction

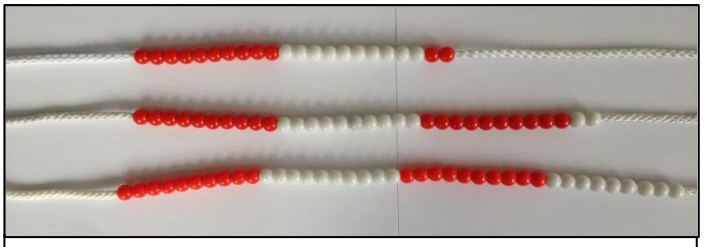
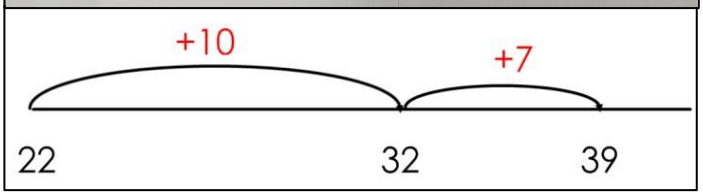
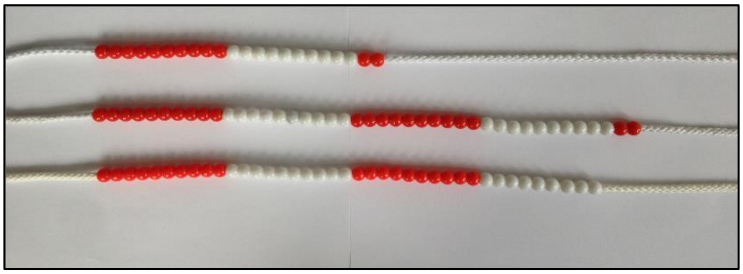
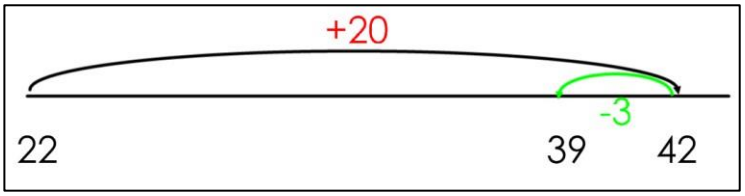
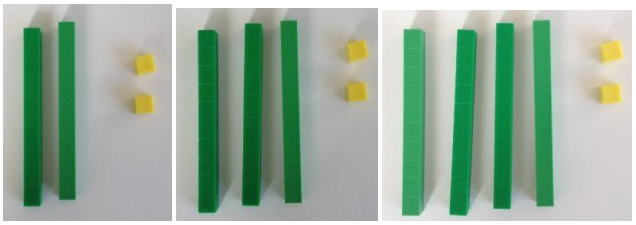
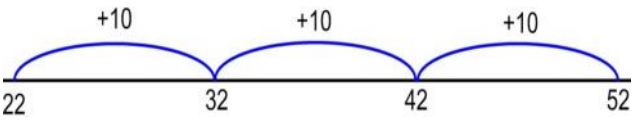
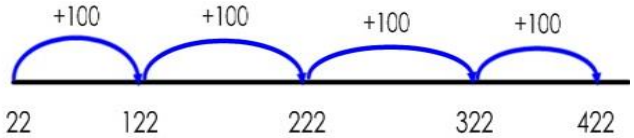
These objectives are explicitly covered through the strategies outlined in this document:

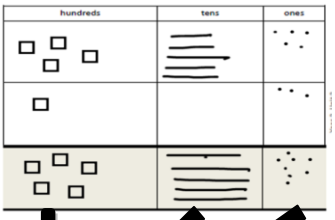
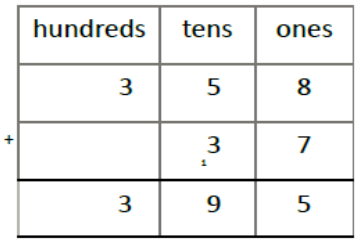
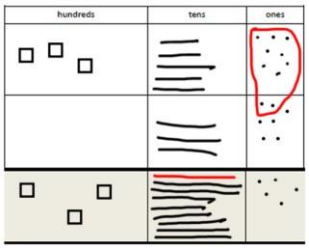
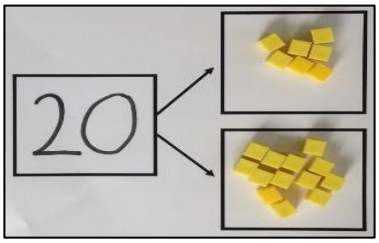
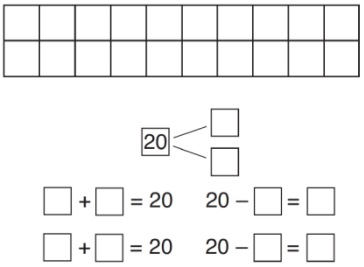
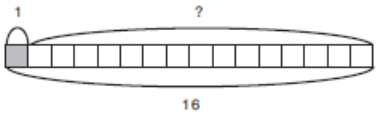
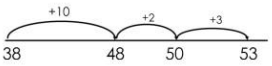
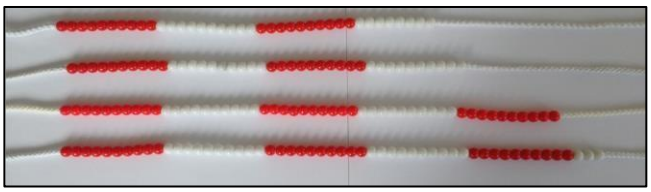
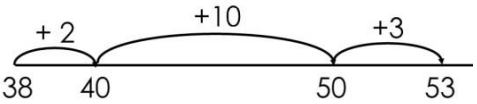
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; 2 two-digit numbers; adding three one-digit numbers.
- Add and subtract numbers mentally, including: a three-digit number and ones; a three-digit number and tens; a three-digit number and hundreds (Year 3).
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Find 10 or 100 more or less than a given number (Year 3).
- Show that addition of two numbers can be done in any order (commutative) but subtraction of one number from another cannot.
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.***
- Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction (Year 3).

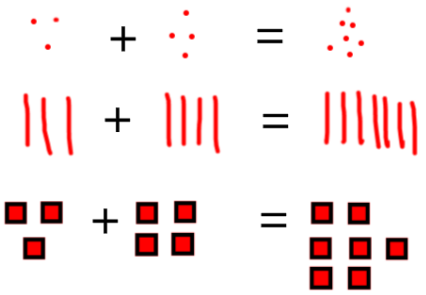
The following objectives should be planned for lessons where new strategies are being introduced and developed:

- Solve problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures; apply increasing knowledge of mental and written methods.
- Solve problems, including missing number problems, using number facts, place value and more complex addition and subtraction. (Year 3)

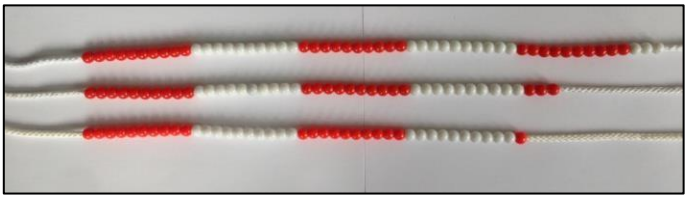
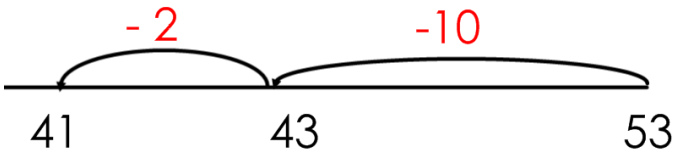
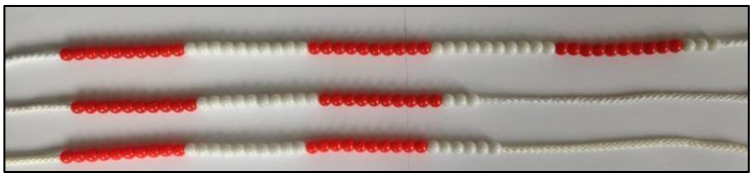
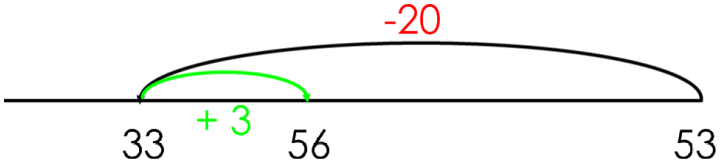
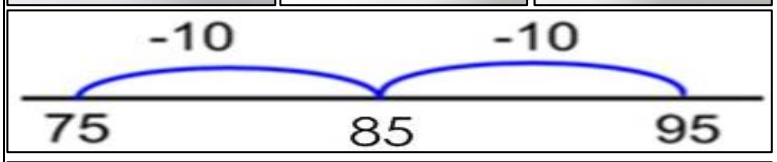
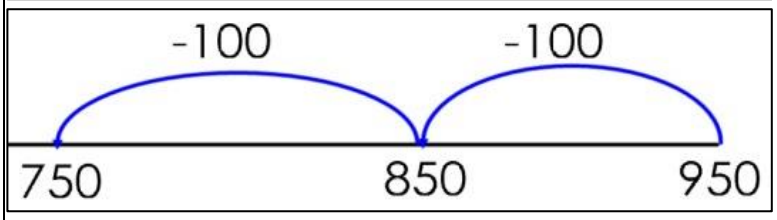
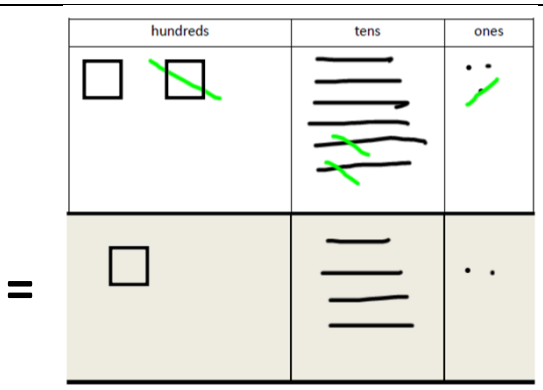
ADDITION – Year 2

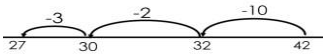
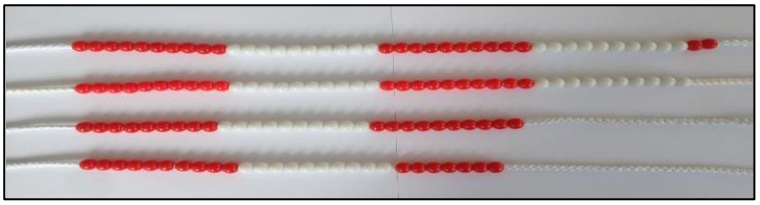
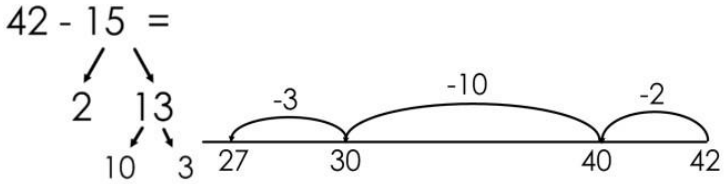
	Strategy and Guidance	Concrete Pictorial Abstract (CPA) Approaches
Step 1	**Partitioning one number, then adding tens and ones** Pupils can choose themselves which of the numbers they wish to partition. Pupils will begin to see when this method is more efficient than adding tens and taking away the extra ones, as shown.	**22 + 17 = 39**  
Step 2	**Rounding one number, then adding the tens and taking away extra ones** Pupils will develop a sense of efficiency with this method, beginning to see when rounding and adjusting is more efficient than adding tens and then ones.	  **22 + 17 = 39**
Step 3	**Counting on in tens and hundreds**	  

Step 4	**Partitioning to add without regrouping** As in Year 1, this is a mental strategy rather than a formal written method. Pupils use the Dienes blocks (and later, images) to represent 3- digit numbers but do not record a formal written method if there is no regrouping.	 **4 5 5 + 1 0 3 = 5 5 8**
Step 5	**Column method with regrouping** Dienes blocks should be used alongside the pictorial representations; they can be placed on the place value grid before pupils make pictorial representations. As in Year 1, the focus for the column method is to develop a strong understanding of place value. See additional guidance on unit pages for extra guidance on this strategy.	 
Step 6	**Part-part-whole** Pupils explore the different ways of making 20. They can do this with all numbers using the same representations. This model develops knowledge of the inverse relationship between addition and subtraction and is used to find the answer to missing number problems.	  □ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1 
Step 7	**Make ten strategy**  How pupils choose to apply this strategy is up to them; however, the focus should always be on efficiency.	 38 + 15 = 2 13 10 3 

Step 8	**Using known facts** Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy.	 3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700

SUBTRACTION – Year 2

	Strategy and Guidance	Concrete Pictorial Abstract (CPA) Approaches
Step 1	**Subtracting tens and ones** Pupils must be taught to partition the second number for this strategy. Pupils will begin to see when this method is more efficient than subtracting tens and adding the extra ones, as shown.	  **53 – 12 = 41**
Step 2	**Subtracting tens and adding extra ones** Pupils must be taught to round the number that is being subtracted. Pupils will develop a sense of efficiency with this method, beginning to identify when this method is more efficient than subtracting tens and then ones.	  **53 – 17 = 36**
Step 3	**Counting back in multiples of ten and one hundred**	  
Step 4	**Partitioning to subtract without regrouping** As in Year 1, the focus is to develop a strong understanding of place value and pupils should always be using concrete manipulatives alongside the pictorial. Formal recording in columns is unnecessary for this mental strategy. It prepares them to subtract with 3-digits when regrouping is required. Please also see additional guidance on unit pages for extra guidance on this strategy.	 **263 – 121= 142**

Step 5	**Column method with regrouping** As in Year 1, the focus for the column method is to develop a strong understanding of place value and pupils should always be using concrete manipulatives alongside the pictorial. Pupils are introduced to calculations that require two instances of regrouping (initially from tens to one and then from hundreds to tens). E.g. 232 – 157 and are given plenty of practice using concrete manipulatives and images alongside their formal written methods, ensuring that important steps are not missed in the recording. **Caution should be exercised on introducing calculations requiring 'regrouping to regroup' (e.g. 204 – 137) ensuring ample teacher modelling using concrete manipulatives and images.**	
Step 6	**Bridging through ten**  How pupils choose to apply this strategy is up to them. The focus should always be on efficiency.	 
Step 7	**Using known number facts** Dienes blocks should be used alongside pictorial and abstract representations when introducing this strategy.	 **8 – 4 = 4** ***leads to*** **80 – 40 = 40** ***leads to*** **800 – 400 = 400**

National Curriculum objectives linked to multiplication and division

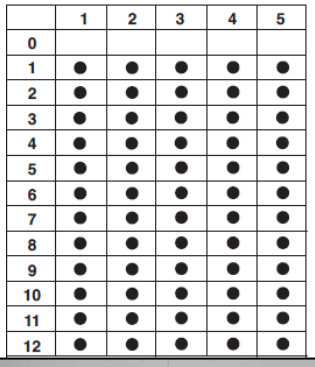
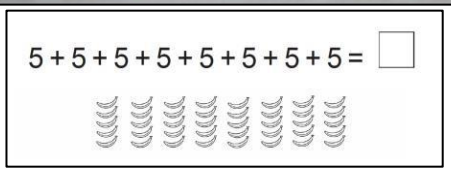
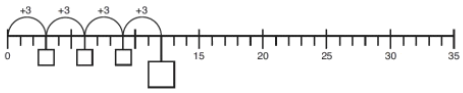
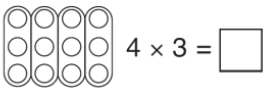
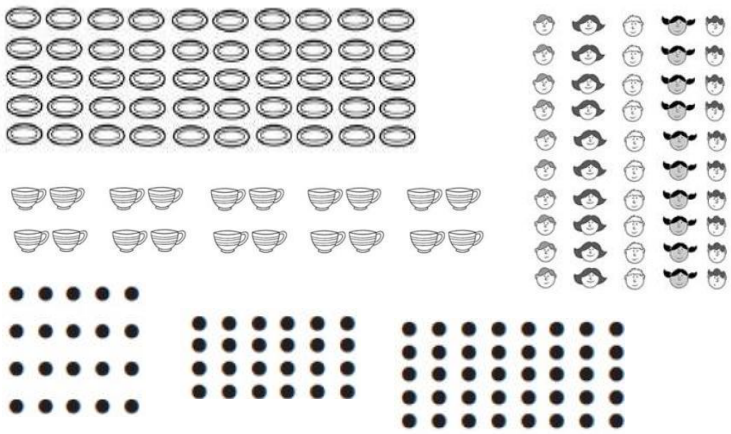
These objectives are explicitly covered through the strategies outlined in this document:

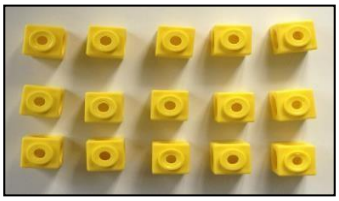
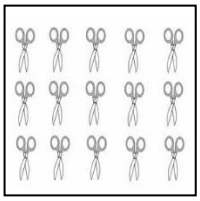
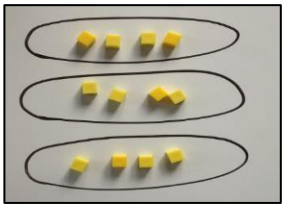
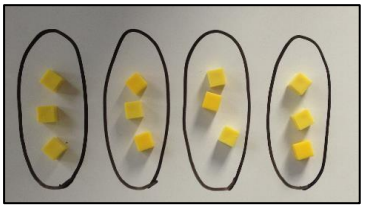
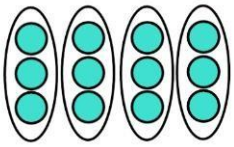
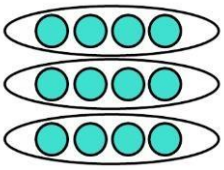
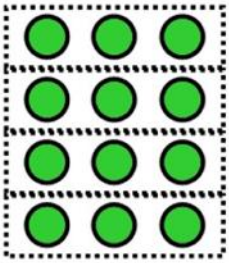
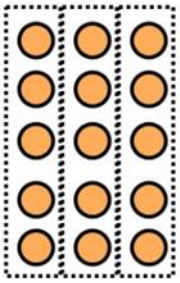
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Recall and use multiplication and division facts for the 3 and 4 multiplication tables (Year 3).
- Show that multiplication of two numbers can be done in any order (commutative) but division of one number by another cannot.

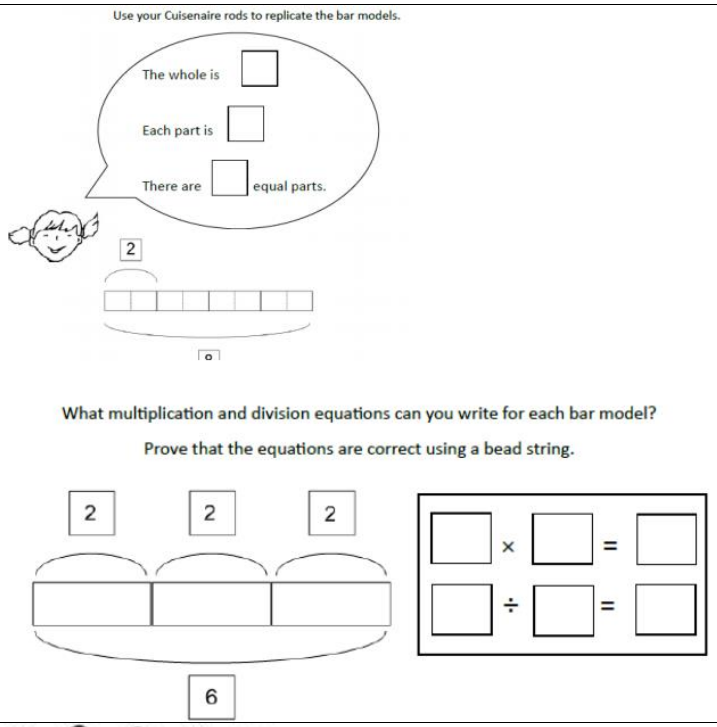
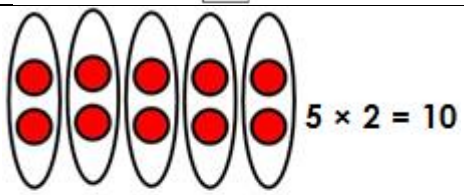
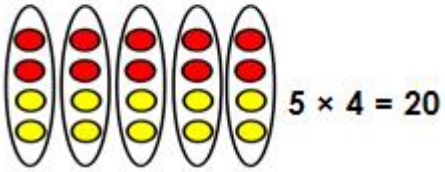
The following objectives should be planned for lessons where new strategies are being introduced and developed:

- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equal (=) signs.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in context.

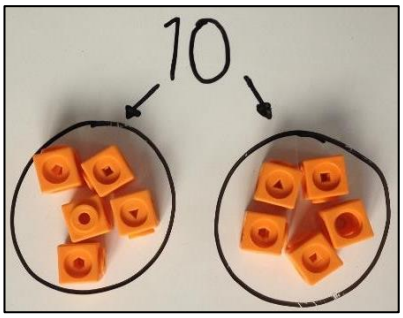
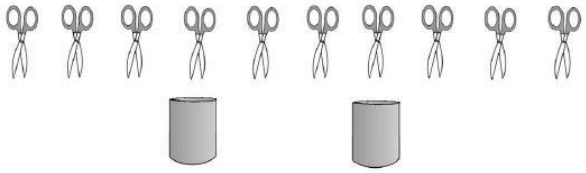
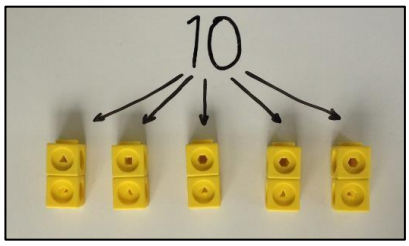
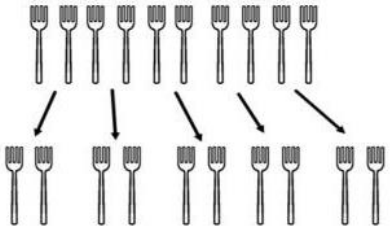
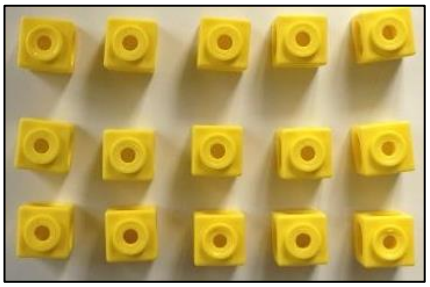
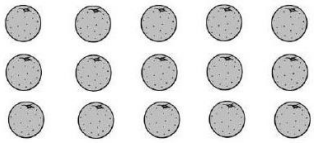
MULTIPLICATION – Year 2

	Strategy and Guidance	Concrete Pictorial Abstract (CPA) Approaches
Step 1	**Skip counting in multiples of 2, 3, 4, 5, 10 from 0** Pupils can use their fingers as they are skip counting, to develop an understanding of 'groups of'. Dotted paper is used to create a visual representation for the different multiplication facts. Each multiplication table has its own template, which is provided during taught units.	 
Step 2	**Multiplication as repeated addition** Pupils will apply skip counting to help find the totals of these repeated additions.	    
Step 3	**Arrays to represent multiplication equations** Concrete manipulatives and images of familiar objects begin to be organised into arrays and, later, are shown alongside dot arrays. It is important to discuss with pupils how arrays can be useful. Pupils begin to understand multiplication in a more abstract fashion, applying their skip counting skills to identify the multiples of the 2x, 5x and 10x tables. The relationship between multiplication and division also begins to be demonstrated.	

Step 4	**Multiplication is commutative** Pupils should understand that an array and, later, bar models can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	 3 x 5 = ☐ 5 x 3 = ☐      **12 = 3 × 4** **12 = 4 × 3**
Step 5	**Bar modelling to represent the parts, the whole and the number of parts in multiplication word problems** Cuisenaire rods can be used to create bar models that represent multiplications.	There are 4 bags of sweets with 3 sweets in each bag. How many sweets are there altogether?  There are 3 school bags with 5 books in each one. How many books are there altogether? 

Step 6	**Use of part-part- whole model to establish the inverse relationship between multiplication and division** This link should be made explicit from early on, using the language of the part-part-whole model, so that pupils develop an early understanding of the relationship between multiplication and division. Bar models (with Cuisenaire rods) should be used to identify the whole, the size of the parts and the number of parts.	
Step 7	**Doubling to derive new multiplication facts** Pupils learn that known facts from easier times tables can be used to derive facts from related times tables using doubling as a strategy. At this stage they double the 2x table facts to derive the 4x table facts.	 

DIVISION – Year 2

	Strategy and Guidance	Concrete Pictorial Abstract (CPA) Approaches
Step 1	**Division as sharing** Here, division is shown as sharing. If we have ten pairs of scissors and we share them between two pots, there will be 5 pairs of scissors in each pot.	 **10 ÷ 2 = 5** 
Step 2	**Division as grouping** Here, division is shown as grouping. If we have ten forks and we put them into groups of two, there are 5 groups.	**10 ÷ 2 = 5**  
Step 3	**Use of part-part-whole model to represent division equations and to emphasise the relationship between division and multiplication** Pupils use arrays of concrete manipulatives and images of familiar objects to find division equations. They begin to use dot arrays to develop a more abstract concept of division.	 15 ÷ 5 = 3 15 ÷ 3 = 5  Write the division equations that the array represents.  20 ÷ 4 = ☐ 20 ÷ 5 = ☐